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Estimating Trend Total Factor Productivity Growth in Canada

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ABSTRACT

This paper presents the methodology used by the Department of Finance to calculate total factor productivity (TFP) at the aggregate level for the commercial sector of the Canadian economy. The Solow residual which underlies the trend TFP estimates is calculated using a constant returns to scale Cobb-Douglas production function with capital and labour as inputs. The labour input is measured as total hours worked in the commercial sector, thereby capturing the decline in average weekly hours over the 1960s and 1970s and reducing cyclical variations in the residual. The capital stock is adjusted for capacity utilization, further reducing cyclical variability in the residual. The trend TFP series is obtained by identifying break points in the cyclically-adjusted Solow residual and estimating trend growth between these points using a spline function. The results are robust to alterations in the break point, depreciation rate, and capacity utilization assumptions.

Our estimates are compared with those obtained by Statistics Canada, the Economic Council of Canada and the OECD. These other studies use the same basic methodology to estimate the Solow residual, but their results are quite different. Differences in results stem primarily from the other studies' use of employment rather than total hours worked, of capital unadjusted for capacity utilization and of Fixed Capital Flows and Stocks rather than Canadian Income and Expenditure Accounts investment data. The other studies generally estimate lower TFP growth in the 1960s and 1970s, because of higher growth in the labour input, and higher TFP growth in the 1980s, because of slower growth in the capital input. Their estimates of the trend TFP growth decline in the 1980s are thus smaller than that obtained by Finance. Nevertheless, all of the studies, including ours, obtain the same key qualitative conclusion that Canadian TFP growth has slowed significantly in the 1970s and the 1980s.

RÉSUMÉ

Ce document présente la méthodologie utilisée par le ministère des Finances pour calculer la productivité totale des facteurs (PTF) pour le secteur commercial de l'économie canadienne. Le résidu Solow qui sous-tend les estimations tendancielles de PTF est calculé à l'aide d'une fonction de production Cobb-Douglas à rendements constants à l'échelle, dont les intrants sont le capital et la main-d'oeuvre. L'intrant main-d'oeuvre est mesuré en heures totales travaillées dans le secteur commercial, ce qui permet de tenir compte du fléchissement du nombre moyen d'heures travaillées par semaine dans les années 60 et 70 et de réduire les variations cycliques du résidu. Le stock de capital est ajusté en fonction de l'utilisation des capacités de production, réduisant encore les variations cycliques du résidu. La PTF tendancielle est obtenue en déterminant les points de retournement du résidu Solow corrigé de ses variations cycliques et en estimant la croissance tendancielle entre ces points en s'assurant qu'il n'y ait pas de discontinuité entre les périodes ("spline

function"). Les résultats ainsi obtenus ne sont pas affectés substantiellement lorsque les hypothèses liées aux points de retournement, au taux d'amortissement et au taux d'utilisation des capacités sont modifiées.

Nos estimations sont comparées aux résultats obtenus par Statistique Canada, le Conseil économique du Canada et l'OCDE. Les études effectuées par ces organismes utilisent essentiellement la même méthodologie que le Ministère pour évaluer le résidu Solow, mais obtiennent des résultats sensiblement différents. La différence entre nos estimations et celles obtenues dans ces études s'explique principalement par le fait que ces dernières: (1) utilisent l'emploi au lieu du nombre d'heures travaillées pour mesurer l'intrant main-d'oeuvre; (2) n'ajustent pas le stock de capital en fonction de l'utilisation des capacités; (3) tirent les données d'investissement de l'enquête des flux et stocks de capital fixe plutôt que des comptes canadiens de revenus et de dépenses. Ainsi, ces études obtiennent généralement des estimations plus faibles de la croissance de la PTF dans les années 60 et 70, en raison de la plus forte croissance de l'intrant main-d'oeuvre, et plus élevées dans les années 80, en raison de la plus faible croissance de l'intrant capital. Leurs estimations du fléchissement de la croissance tendancielle de la PTF dans les années 80 sont donc moins importantes que celle obtenue par le ministère des Finances. Il n'en demeure pas moins que toutes ces études, incluant la présente, obtiennent la même conclusion clé d'un point de vue qualitatif: la croissance de la PTF au Canada a ralenti considérablement au cours des années 70 et 80.

1. Introduction

Productivity growth can be measured in a variety of ways. Labour productivity, defined as output per worker or per person-hour, is perhaps the most familiar measure of productivity. Labour productivity is an important measure since it indicates the potential for non-inflationary increases in labour compensation. It is often misinterpreted, however, as a measure of efficiency, which it is not since it cannot distinguish between overall efficiency gains and gains arising from an increase in the use of other inputs. Total factor (or multifactor) productivity growth is a much better measure of overall efficiency gains since it measures an economy's ability to obtain increasing amounts of real output from given levels of all factor inputs, and not just labour. It captures the effects of improved organization in the workplace, enhanced skills of the labour force, the adoption of new technologies and the availability and efficiency of infrastructure. In the standard neoclassical growth model, total factor productivity (TFP) growth is actually the underlying determinant of long-run labour productivity growth.[1]

TFP growth is a key determinant of the rate at which an economy can provide its citizens with improved living standards. The goal of this paper is to explain the comprehensive approach to estimating TFP growth in use at the Department of Finance and how it differs from that of other organizations such as Statistics Canada, the Economic Council of Canada and the Organization for Economic Cooperation and Development (OECD).

Most studies agree that there has been a trend decline in Canadian TFP growth since the 1960s and 1970s. A number of studies, however, show a pick-up in TFP growth in the latter part of the 1980s, while the Department estimates that TFP growth was flat over this period. Differences in quantitative results, which are especially evident in estimates of TFP growth over short intervals of time, arise from differences in both methodology and data sources. Nevertheless, the qualitative conclusions of a general slowdown are similar.

This study finds that the decline in trend TFP growth was not exclusive to Canada, but was also experienced by the United States. Other researchers have found the same broad pattern for other G-7 countries. Most G-7 countries, however, have enjoyed some degree of recovery in TFP growth in the 1980s, while Canada has not.[2]

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1. In the Solow (1956) growth model, the steady-state golden rule growth path is characterized under Harrod-neutral labour-augmenting technical change by a capital-labour ratio which grows at the rate of labour productivity. Along the optimum growth path, both of these growth rates are uniquely determined by the underlying rate of technological efficiency (or TFP) growth.

2. Englander and Mittelstadt (1990).

The remainder of the paper is organized as follows. The next section outlines the methodology underlying the Department's TFP estimates. It details the choice of data involved in estimating trend TFP growth, and raises some associated methodological issues. Section 3 presents the empirical results for Canada and explores the sensitivity of the results to alternative assumptions. Section 4 compares the estimates for Canada to those obtained by Statistics Canada, the Economic Council of Canada and the OECD. Section 5 compares trend TFP growth in Canada to that in the United States using identical methodologies. The final section draws conclusions.

2. Methodology Underlying Total Factor Productivity Estimates

2.1 Defining Total Factor Productivity

Conceptually, TFP simply measures the overall efficiency of the entire economy. Ideally, it should take account of improvements in the efficiency of every type of input, not just of labour alone. The inputs that are most often considered in the literature, and by the Department of Finance, are capital and labour. In some cases, however, energy and raw materials are also considered as inputs.[3]

In his seminal article on technical change, Solow (1957) calculated TFP growth residually after specifying a production technology for the economy, i.e. a relationship between output and inputs.[4] Measurements obtained in this way are called the Solow residual. The exact formulation of the economy's technology is obviously a matter of choice. The assumption made by the Department of Finance and many other researchers is that aggregate technology can be reasonably and simply represented by a constant returns to scale Cobb-Douglas production function.[5]

Several assumptions are implicit in these choices. The Cobb-Douglas production function used by Finance is specified so that there are constant returns

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3. For example, Helliwell et al. (1985) and Torres et al. (1990) consider energy while Jorgenson (1988) considers energy and raw materials.

4. An alternative approach is to estimate the cost function which is dual to the production function (see for example, Morrison (1986), Berndt and Fuss (1986) and Slade (1989)). The cost function gives the minimum cost of producing a particular output given a technology and a vector of input prices. Technical change can be measured residually as the change in average cost not accounted for by the changing contributions of input prices.

5. Some, however, choose a CES production function. See for example Torres et al. (1990).

to scale in factor inputs. There are thus no increasing returns to scale, internal to firms or through externalities or "thick" market effects. The Solow approach assumes that all input prices are determined competitively and that all firms choose their inputs to maximize profit.

The Solow approach has not gone unchallenged. As Hall (1990) points out, these assumptions imply that the Solow residual should be uncorrelated with any variable that would be uncorrelated with "true" TFP if the latter could be directly observed. Using U.S. data, he concludes that this is not the case, calling into question the validity of the measure and the assumptions concerning constant returns to scale, perfect competition and non-existence of externalities. Baily (1990), however, criticises the econometric tests that Hall uses to reach these conclusions. Baily advocates the Solow measure and asserts moreover that neither imperfect competition nor increasing returns would in any case invalidate the analysis of longer-run growth trends.

In a Cobb-Douglas production function, TFP is the technological efficiency term "Q":

$$Y = QL^{\alpha}K^{(1-\alpha)} \qquad (2.1)$$

where Y is real output, L is the labour input, α is labour's share in income and K is the capital stock. Q is the residual contribution to output that ensures that the production function holds. In the absence of special adjustments, it will be affected by purely cyclical movements as well as changes in long-run efficiency. The amount of capital in use, for example, will vary substantially over the business cycle even though the capital stock in place may not. This will lead to large cyclical movements in the capital-output ratio (measured in terms of the available capital stock), and thus to large cyclical movements in Q. Winston (1974) has pointed out that transactions costs, lumpiness, and wage shift differentials can reconcile profit maximization with a ratio of the flow of capital services to the capital stock which is both variable and at times less than unity. Adjusting the available capital stock by a utilization rate provides an estimate of the actual capital input, which should significantly reduce the cyclical fluctuations inherent in Q and help identify longer-run trends in TFP. This is accomplished by introducing a capacity utilization adjustment, λ:[6]

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6. Capacity utilization adjustments in the context of the dual cost function approach are advised by Slade (1989), Berndt and Fuss (1986), Hulten (1986) and Morrison (1985). Hulten dispenses with the flow of services concept and defines intensity of use solely in terms of the amount of the variable factor applied to the quasi-fixed factor. In this context the capacity adjustment simply takes the form of accounting for the quasi-fixed factor rents when calculating factor shares as described in footnote 10. Berndt and Fuss, however, recommend adjusting capital values in a manner which can be considered a dual representation of an adjustment to the capital stock for variations in the flow of services.

$$Y = QL^{\alpha}(\lambda K)^{(1-\alpha)} \qquad (2.2)$$

The cyclically-adjusted Solow residual, Q, is thus defined as:

$$Q = Y/(L^{\alpha}(\lambda K)^{(1-\alpha)}) \qquad (2.3)$$

This residual is a cyclically-adjusted measure of TFP. Movements in this measure of the Solow residual can arise from technological advances, improved quality of inputs, intersectoral reallocation of resources, relative price shifts, etc., but should not arise from cyclical fluctuations in the inputs.[7]

2.2 Choice of Data and Parameterization

Obtaining an estimate of the Solow residual (equation 2.3) involves many choices with respect to measures of output, the factor inputs and the parameters of the production function. A discussion of the choices made in Department of Finance modelling and related issues follow. A comparison with approaches used in other studies is made in Section 4.

2.2.a Output

The level of aggregation chosen is the commercial sector, which measures the portion of the economy that can be reasonably described as behaving in a cost-minimizing fashion. Commercial sector output at factor cost is obtained by subtracting consumer expenditures for rent, total government wage expenditures, and capital consumption allowances for the federal, provincial, municipal and hospital sectors from gross domestic product (GDP) at factor cost, which is equal to GDP at market prices net of indirect taxes and subsidies. Rent is excluded because it is largely imputed, while the government sector is excluded because its output is estimated by inputs, and hence its measured productivity growth is zero by construction. This measure is based on unpublished data for the government sector from the Canadian Income and Expenditure Accounts (CIEA).

Chart 2.1 compares real GDP at factor cost in the commercial sector, the business sector (as defined by Statistics Canada) and the total economy. Commercial sector output is consistently smaller than business sector output, as it excludes consumer expenditure on paid and imputed rent. Rent represents the flow

7. A thorough discussion on this can be found in Abramovitz (1991).

of services from residential structures which are not included in the capital input measure. All three series exhibit very similar profiles.

2.2.b Labour

The labour input is measured as total hours worked in the commercial sector. This series is obtained from unpublished data constructed for the Department of Finance by Statistics Canada using Labour Force Survey (LFS) data These data are smoothed by taking a centred moving average[8]. The household-based LFS data are used because they have greater internal consistency than the establishment-based Survey of Employment Payroll and Hours (SEPH).

Using total hours worked is preferable to simply using the total number of employees for two reasons. First, this measure accounts for the declining trend in average hours worked per week through the 1960s and 1970s, which would otherwise be interpreted as a decrease in the residual (Charts 2.2 and 2.3). Second, since employers may alter the work week over the business cycle rather than adjust employment levels, this measure avoids embodying a spurious cyclicity in the residual.

The use of hours may not completely eliminate the cyclicity inherent in the residual, however, because of cyclical variations in labour hoarding or the quality of work effort (Hall (1990)). Unfortunately, data needed to adjust for these factors are unavailable. Measuring the labour input in hours worked, however, ensures that more of the cyclicity of the labour input is accounted for than if labour were measured as employment alone.

2.2.c Capital

The capital stock used in calculating the Solow residual is the end-of-year net stock in all industries less the stock of the federal, provincial and municipal governments, hospitals, and schools. The net capital stock excludes depreciated capital and is therefore a better measure of capital's productive ability than the gross stock.

The capital stock, K, is measured as a bundle of machinery and equipment, K_{me}, and non-residential structures, K_{nr}:

$$K = K_{me}^{\beta} K_{nr}^{(1-\beta)} \qquad (2.4)$$

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8. The series is smoothed by taking a 9-quarter centred moving average.

CHART 2.1
GDP AT FACTOR COST

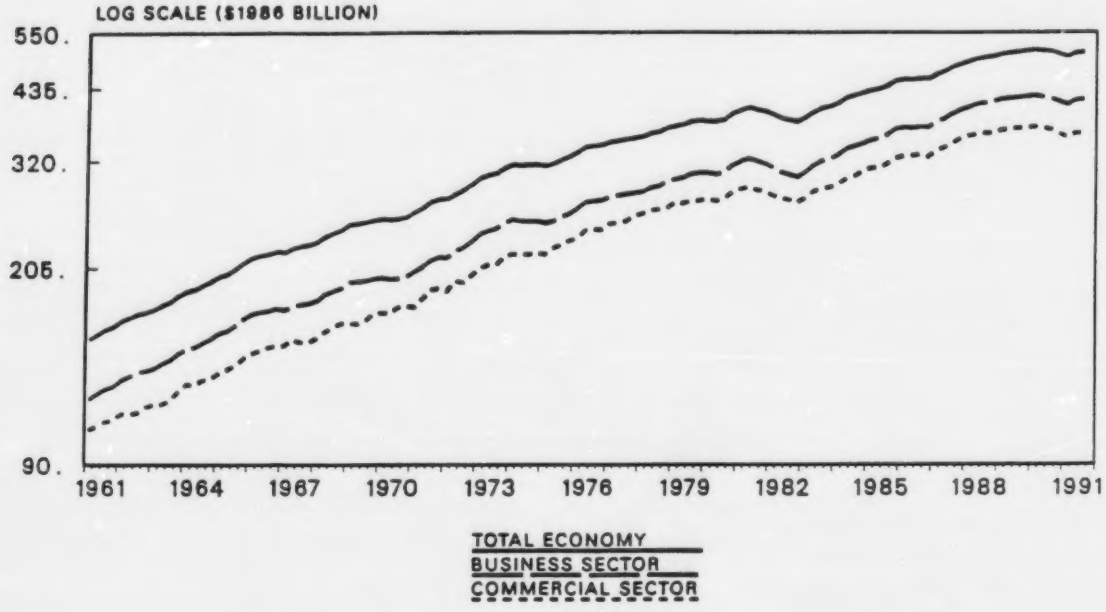


CHART 2.2
AVERAGE WEEKLY HOURS IN THE COMMERCIAL SECTOR

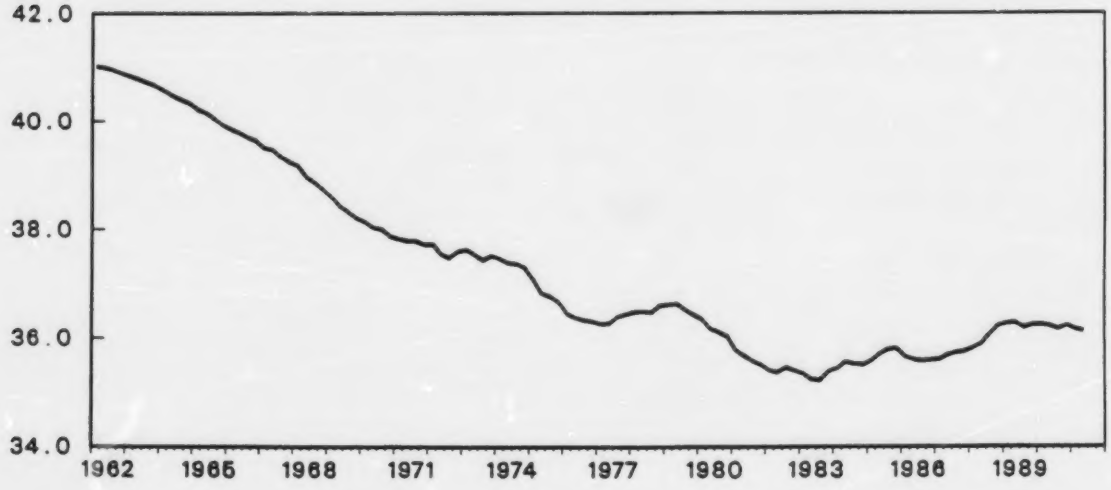


CHART 2.3
COMMERCIAL SECTOR LABOUR INPUT INDICES
(1962:1 = 1.0)

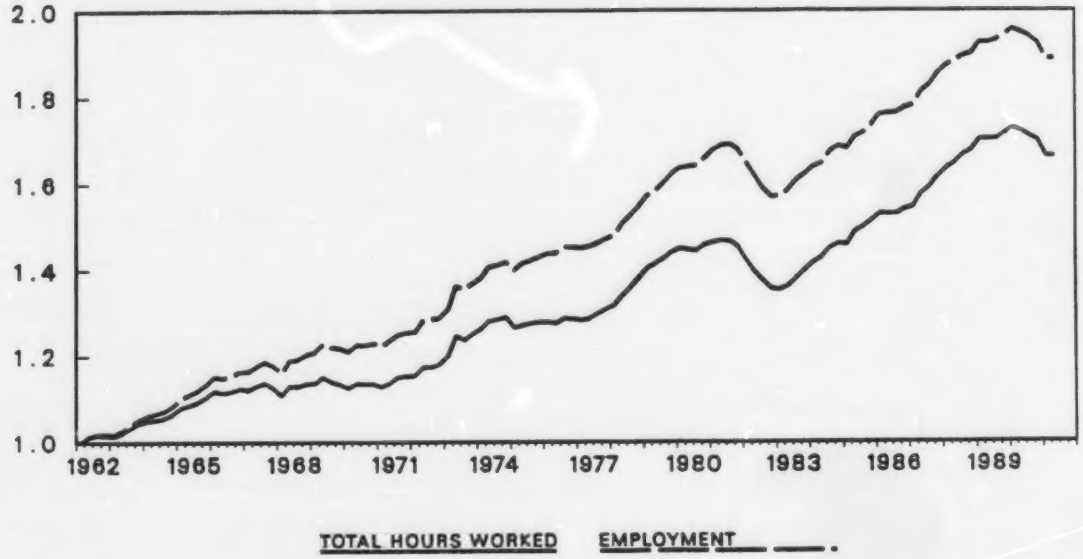
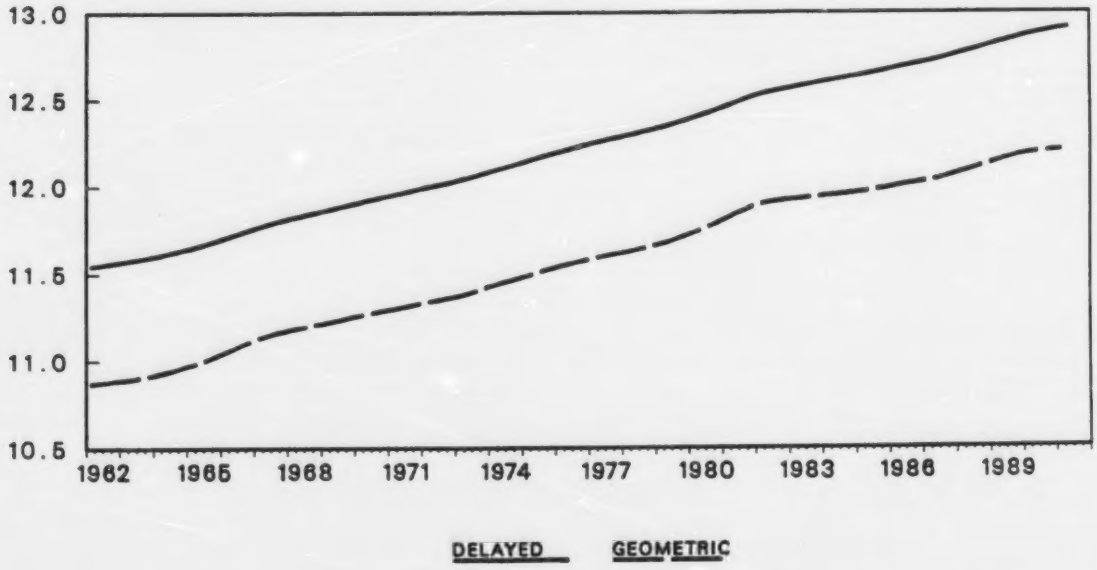


CHART 2.4
CAPITAL STOCK
(LOG)



where β, estimated to be 0.4, is the average of the ratio of machinery and equipment income to the sum of the machinery and equipment and non-residential structures income. The methodology used to calculate this share is detailed in Annex A.

An end-of-year net capital stock is created for each category using the following identity:

$$K_{t,i} = (1-\delta_{t,i})K_{t-1,i} + I_{t,i} \quad ; \quad i = me, nr \qquad (2.5)$$

where δ is the depreciation rate and I is business sector gross investment from the Canadian Income and Expenditure Accounts (CIEA). A starting value (1947:1) for each capital stock is taken from Statistics Canada's Fixed Capital Flows and Stocks (FCFS). The depreciation rate is imputed from FCFS investment data and the corresponding estimates of the capital stock derived using the **delayed** depreciation assumption.

The revised FCFS data provide capital stock series based on straight-line, delayed or geometric depreciation assumptions.[9] For a growing economy, a delayed depreciation rate yields a higher net capital stock than geometric depreciation (Chart 2.4). The evidence for each depreciation assumption is mixed. Koumanakos and Hwang (1988), on the basis of asset prices, conclude that the pattern of economic depreciation is close to the geometric form in the manufacturing sector, but that it is close to straight-line form in the non-manufacturing sector. Oliner (1989) and Novin (1989) both argue that, while a geometric depreciation rate may be more appropriate for calculating the market value of capital, a capital stock based on a delayed depreciation rate may be more appropriate when measuring its productive use.[10] The basic argument is that asset prices reflect the "irreversible" nature of some elements of capital investment, yet the productivity of capital in place is not diminished to a corresponding degree.

Equation 2.5 will yield a different capital stock series than the Statistics Canada series because of substantial differences between the CIEA and FCFS investment flows. The differences in the nominal gross investment data are small, and result primarily from CIEA definitional adjustments that are not adopted by the

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9. Statistics Canada publishes three set of depreciation rates by industry: straight-line, delayed and geometric. Straight-line depreciation assumes that an asset depreciates by a constant amount over its entire economic life. In contrast, delayed depreciation assumes that an asset depreciates slowly initially, but that the depreciation rate increases as the asset reaches the end of its life. In the case of geometric depreciation, it is assumed that the asset depreciates by a constant percentage over its life.

10. As shown in Section 3.3, the choice of depreciation methods does not in practice significantly alter the pattern of growth in trend total factor productivity.

FCFS. The differences in the constant-dollar investment data are much more pronounced, amounting to as much as 40 per cent in the early 1970s; differences since 1980 have been much smaller. This is primarily the result of different types of aggregation used in the deflation process. Further details are included in Annex B.

As mentioned previously, simply measuring the capital input as the available capital stock will incorporate cyclical variation in the utilization of capital in the Solow residual. To account for cyclical fluctuations which might otherwise be falsely interpreted as fluctuations in trend TFP, the capital stock data are adjusted for capacity utilization. This adjustment is made using the Bank of Canada's capacity utilization index. Since the Bank's capacity utilization index only covers goods-producing industries, it is adjusted to account for the lower variability of the capital-output ratio in the service industry over the business cycle. Details of this adjustment are presented in Annex C.

2.2.d Production Function Parameters

From equation 2.3, it follows that TFP growth is equal to the growth in output minus the weighted growth rates of the capital and labour inputs:

$$\Delta\ln(Q) = \Delta\ln(y) - \alpha\Delta\ln(L) - (1-\alpha)\Delta\ln(\lambda K) \qquad (2.6)$$

Assuming perfect competition (so that labour is paid its marginal product) and constant returns to scale (so that payments to factors exhaust all revenues)[11], α is labour's share in total revenues:

$$\alpha = wL/(pY) \qquad (2.7)$$

where w is the marginal product (in value) of labour and p is the price level. As shown in Chart 2.5, labour's share of total revenues varies significantly from period to period (although longer-run trends are not clear), which would seem to imply that α should also vary. The Törnqvist index is a natural candidate since it is a 2-period moving average of factor shares. Diewert (1976) shows that this is a

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11. This reflects underlying assumptions regarding the relevance of rents accruing to quasi-fixed capital. With capital and labour variable, the more general form of (2.6) is:

$$\Delta\ln(Q) = \Delta\ln(y) - \alpha\Delta\ln(L) - \psi\Delta\ln(\lambda K) \qquad (2.6')$$

where $\psi = uK/(pY)$, and u is the user cost of capital. As pointed out by Berndt and Fuss (1986) and Hulten (1986), use of (2.6') will be erroneous when rents are accruing to quasi-fixed capital. Hulten suggests that given the assumption of constant returns to scale, the shadow price of capital can be calculated residually by subtracting labour income from output.

discrete time approximation to the continuous time index for the Cobb-Douglas production function.

If Törnqvist weights are employed, however, this implicitly assumes that factors are paid their marginal product (in value) in each period. If workers in a particular period are paid more than their marginal product, however, then α would be overestimated, and vice versa. Time-varying Törnqvist weights may therefore be less appropriate than fixed (average share of labour income in total income) weights if perfect competition holds on average but not in every period. Moreover, since labour's share in total revenues is influenced by cyclical factors, fixed weights avoid introducing unwarranted cyclicity in the Solow residual. Fixed weights are therefore used in this study; the estimate for α is 0.637.[12]

2.3 Estimation of Trend Total Factor Productivity

Even with the cyclical adjustment, the Solow residual exhibits a considerable amount of short-term noise, making average growth rates over a specific time frame very sensitive to the choice of end points, and thus potentially misleading. Moreover, the existence of data measurement errors and short-run rigidities implies that the production function will only hold on average, rather than in every period. Longer-run trends in the residual are therefore more revealing about underlying developments in true TFP.

As demonstrated by Zohar and Luski (1987), the growth in the Solow residual between two periods is sensitive to the method used to identify underlying trends. Methods using averages of end points, for example, are sensitive to the choice of sample period. Trend TFP growth is estimated in this study using a regression-based approach. This method has an advantage over simple averages based on end-points, as it considers all of the data throughout the sample period.

To generate *trend* TFP growth, the cyclically-adjusted Solow residual is visually examined for break points -- points where the growth rate appears to have undergone a lasting change (Chart 3.1). Trend growth is assumed to be constant between break points. A spline function is then fitted to the cyclically-adjusted Solow residual to estimate trend TFP; linear segments represent periods where trend TFP growth is constant, break points represent times at which trend TFP growth

12. This is measured as the average share of commercial-sector labour income in commercial sector GDP at factor cost over the sample period.

CHART 2.5
LABOUR'S INCOME SHARE IN COMMERCIAL GDP

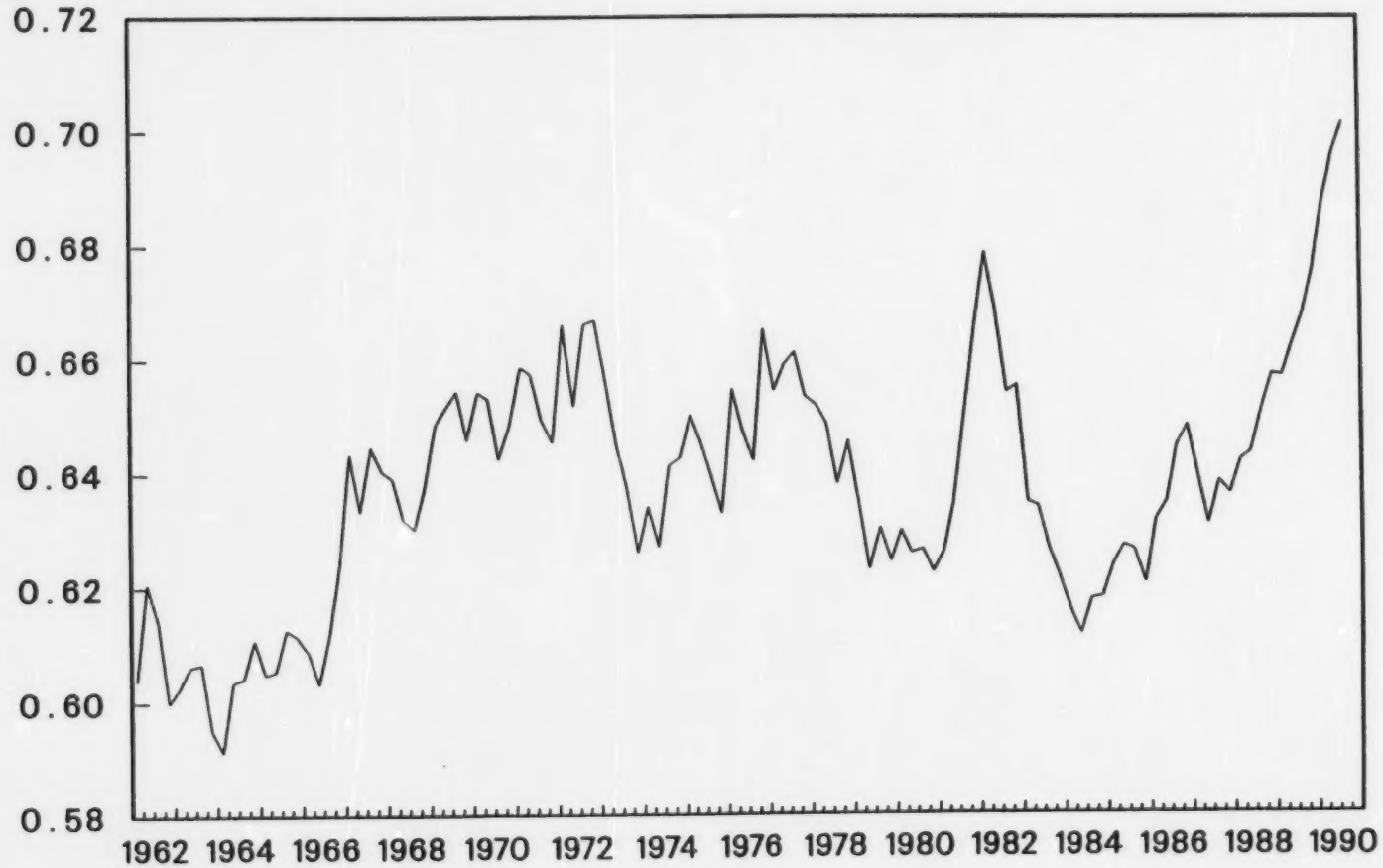


CHART 3.1
SOLOW RESIDUAL
(NATURAL LOG)

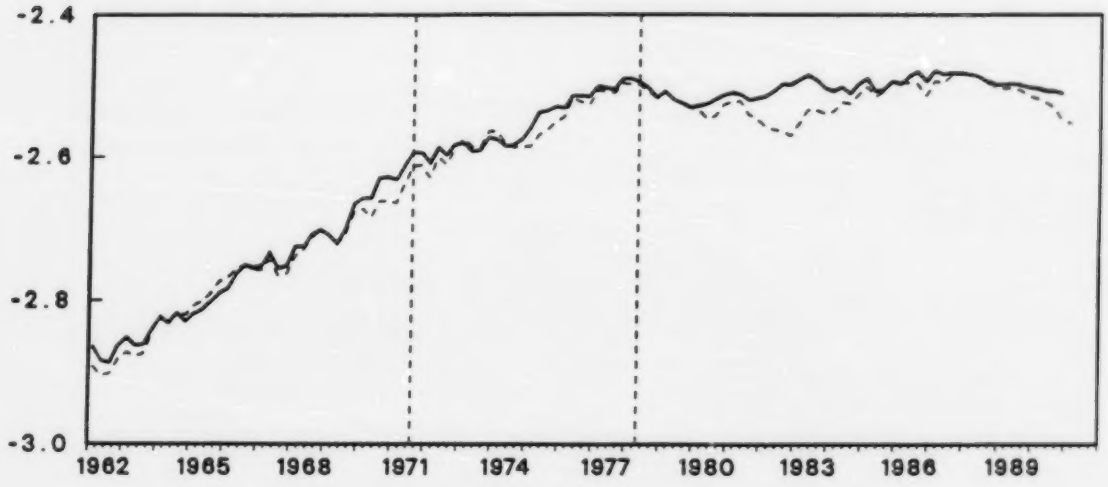
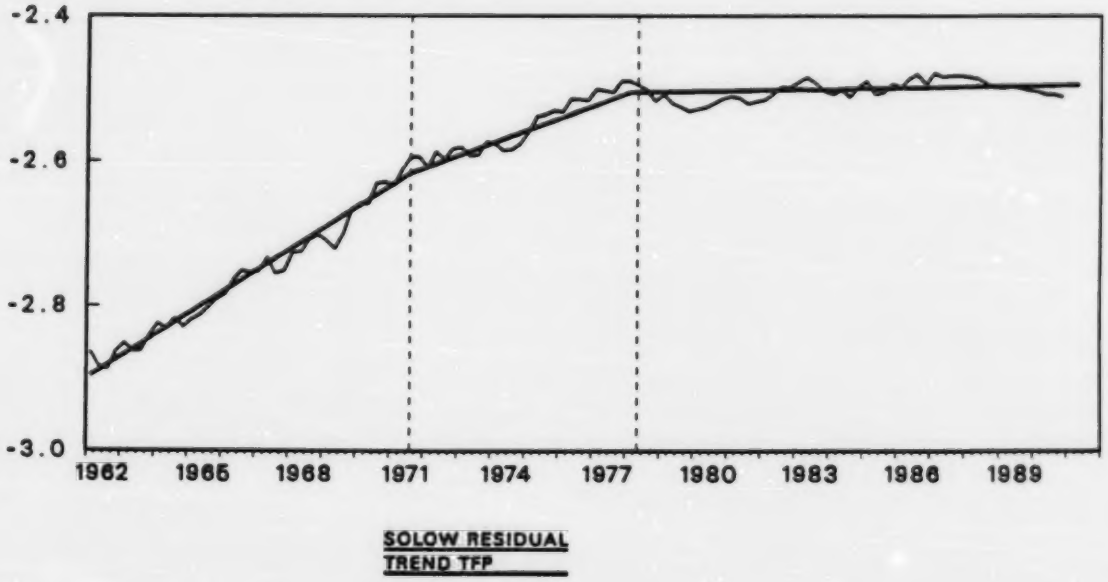


CHART 3.2
TREND TOTAL FACTOR PRODUCTIVITY INDEX
(NATURAL LOG)



changes.[13] In general, if there are breaks in the series at times $T_1, T_2, \ldots, T_i, \ldots$ ($T_0 = 0$), then trend TFP (Q_u) may be written as the spline function:

$$Q_u = \exp[\sum_{i=0}^{J}(\beta_0 + \sum_{i=0}^{I}(\beta_i - \beta_{i+1})T_i + \beta_{i+1}T)D_{i+1}] \qquad (2.8)$$

where the dummy variables D_i are defined:

$$D_{i+1} = \begin{cases} 1 \; \forall \; T_i \leq T < T_{i+1} \\ 0 \; \forall \; T < T_i, \; T \geq T_{i+1} \end{cases}$$

and T_J is the terminal point of the series. The quarterly growth rate of trend TFP for an interval $[T_i, T_{i+1})$ is β_{i+1}.[14]

3. Empirical Results

3.1 Trend Total Factor Productivity Growth

Examination of the cyclically-adjusted Solow residual shown in Chart 3.1 suggests break points in the fourth quarter of 1971 and the second quarter of 1978. The latter is the key break point. Prior to this, trend TFP growth had been strong; since then it has been close to zero.

Without the cyclical adjustment, the Solow residual would probably suggest choosing two additional break points: the first quarter of 1983 and the first quarter of 1987. This would lead to estimates of increased TFP growth over the 1983 to 1987 period and negative TFP growth over the 1987 to 1990 period. Given the severity of the 1981-1982 recession, actual capital in use was in all likelihood far less than the available capital stock. In the absence of a capacity adjustment, the capital input would thus be seriously overestimated, and the Solow residual underestimated, during the recession. This would cause an apparent rise in trend TFP growth once the recovery was underway and utilization rates returned to normal. Similarly, after 1987, utilization of capital in the goods-producing industries was declining. Failure to account for this would result in an underestimation in the growth of the Solow residual. This demonstrates the importance of distinguishing between improvements in **trend** TFP growth and

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13. There are other smoothing methods that could be used. One such method is the Hodrick-Prescott filter which, in practice, yields very similar results to the spline function.

14. The spline function defined by equation 2.8 ensures that trend segments intersect at break points.

improvements observed in the Solow residual growth that are generated by a cyclical recovery or downturn in the economy.

Creating a time index so that T = 1 represents the second quarter of 1962 (the first data point) and using the above break points yields the following specification for equation 2.8:

$$\begin{aligned}\log(Q_m) = {} & (\beta_0+\beta_1 T)D_1 \\ & +(\beta_0+(\beta_1-\beta_2)T_{1971:4}+\beta_2 T)D_2 \\ & +(\beta_0+(\beta_1-\beta_2)T_{1971:4}+(\beta_2-\beta_3)T_{1978:2}+\beta_3 T)D_3 \end{aligned} \qquad (2.9)$$

Trend TFP growth is calculated by estimating the coefficients (the β's) in equation 2.9 and obtaining the fitted values (Chart 3.2). Annual trend TFP growth is estimated to have been 2.9 per cent between 1964:1 and 1971:3, 1.7 per cent between 1971:4 and 1978:1, and 0.1 per cent between 1978:2 and 1990:4. Regression results are reported in Appendix D.

3.2 Decomposition of the Solow Residual and Trend TFP

Table 3.1 presents a detailed breakdown of the calculation of the average growth of the Solow residual adjusted for capacity utilization and of estimated trend TFP growth between the break points. Over the first period, real commercial GDP grew 5.6 per cent with labour and capital accounting for only 1.5 percentage points. If commercial employment were used as the labour input instead of hours worked, labour's contribution would have been overestimated by 0.6 percentage points in this period and growth of the Solow residual underestimated 0.6 percentage points. Similarly, if the capital input was unadjusted for utilization, the average growth rate of the Solow residual would have been underestimated by 0.2 per cent. The second period illustrates a similar pattern.

Over the 1978 to 1990 period, the impact of adjusting for hours worked and capital utilization is less pronounced. In fact, the average growth in the Solow residual over this period would have been underestimated by only 0.1 percentage point if capital were not adjusted for capacity utilization. Examining the decomposition of the Solow residual over the 1983 to 1990 period, however, reveals the significance of input measurement. Specifically, if commercial employment were used as the labour input instead of hours worked, labour's contribution would have been underestimated by 0.2 percentage points and growth in the Solow residual underestimated by the same amount. If the capital input was unadjusted for utilization, the average growth rate of the Solow residual would have been overestimated by 0.5 percentage points.

Table 3.1
Decomposition of the Solow Residual
(average annual per cent changes)

	1964-1971	1972-1978	1979-1990	1983-1990
Real Commercial Output	5.6	5.4	3.0	3.7
less contributions of:				
Labour	0.8	1.3	1.4	1.7
Commercial Employment	1.5	1.7	1.4	1.5
Hours Per Week	-0.6	-0.4	0.0	0.2
Capital	1.6	2.2	1.7	2.1
Machinery and Equipment	0.7	0.9	1.0	1.0
Non-Residential Construction	1.1	0.9	0.8	0.6
Capacity Utilization	-0.2	0.4	-0.1	0.5
equals:				
Solow Residual adj. for cap.[2]	3.1	1.9	-0.1	0.0
(Trend Total Factor Productivity)	2.9	1.7	0.1	0.1

Note: Numbers may not add due to rounding.
(1) Averaging periods are chosen to correspond to break points.
(2) Average of Solow residual is not identical to trend growth.

3.3 Invariance Properties of the Cyclically-Adjusted Solow Residual

As mentioned previously, the constant returns to scale and perfect competition assumptions imply that variables that affect output but not total factor productivity should not be correlated with the Solow residual. Hall (1990) found that his estimates of the U.S. Solow residual did not pass this invariance test. On this basis, he questioned the validity of the measure and its underlying assumptions (perfect competition and constant returns to scale). The results of similar invariance tests for the Canadian Solow residual are reported in Annex E. These results show that, in contrast to the findings of Hall, this does not seem to be a problem with the cyclically-adjusted Canadian Solow residual. The unadjusted Canadian Solow residual does not, in contrast, generally pass the invariance tests, indicating the importance of applying the capacity utilization adjustment to the capital stock.

3.4 Sensitivity to Alternative Assumptions

The estimates of trend TFP incorporate specific choices about the capital stock, with respect to the nature of capital depreciation and the investment data, and break points. To assess the sensitivity of the trend TFP growth estimates for Canada, these assumptions are re-examined in the following experiments.

3.4.a Capital Depreciation Rate

With respect to the capital stock data, trend TFP is reestimated using a revised capital stock series that embodies the geometric depreciation assumption.[15] If the delayed depreciation assumption tends to overestimate the capital stock at an increasing rate, it should lead to an underestimation of trend TFP growth. As shown in Chart 2.4, the capital stock bundle used in the production function is significantly different in level terms under different depreciation assumptions. As would be expected for a growing economy, the delayed depreciation capital stock grows at a slightly faster rate than the geometric depreciation series. Given that the capital stock carries a weight of 0.363 in the production function, however, the difference in capital growth rates does not significantly alter the growth rate of the Solow residual (Chart 3.3). Specifically, the growth rates over the 1960s and 1970s are unchanged, while the growth rate over the 1980s is increased slightly to 0.2 per cent per year.

3.4.b Investment Flow Data

As mentioned in section 2.2.c, the capital stock data differs depending on which investment series (CIEA or FCFS) is used. The sensitivity of the estimate of trend TFP growth to this choice is shown in Table 3.2. Growth over the second and third break periods is 0.2 and 0.3 percentage points higher, respectively, with the capital stock consistent with the FCFS investment data. Both estimates, however, show a dramatic decline in trend TFP growth over the 1978 to 1990 period.

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15. The revised capital stock series is constructed using geometric depreciation assumption and the same methodology outlined in Section 2.2.c.

CHART 3.3
SOLOW RESIDUAL INDEX
(1962:1 = -1.0, LOGARITHMIC SCALE)

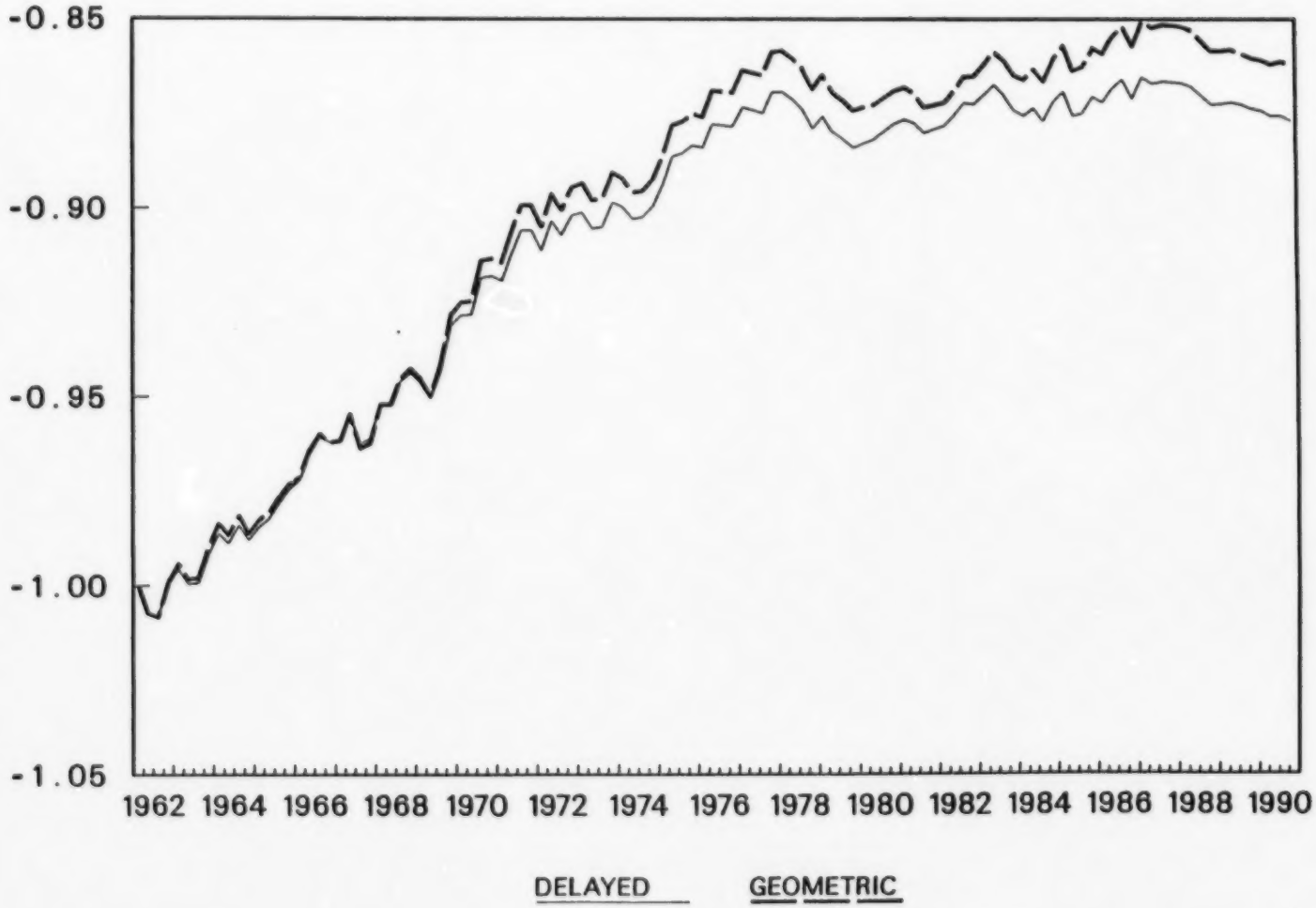


Table 3.2
Trend TFP Growth with Alternative Capital Stock Measures
(per cent)

Capital stock consistent with:	CIEA	FCFS
1964:1-1971:3	2.9	2.9
1971:4-1978:1	1.7	1.9
1978:2-1990:4	0.1	0.4

3.4.c Break Points

With respect to the choice of break points, it is important that the trend TFP growth rate estimates be robust to reasonable alternatives. To test this, the last break point is moved forward and backward four quarters and an alternative smoothing procedure, the Hodrick-Prescott (HP) filter is applied to the cyclically-adjusted Solow residual. The regression results tabled in Annex F show that the growth rates remain virtually unchanged as the break point is altered. Chart 3.4 shows that the spline-function and HP filter based trend TFP series are very similar.[16] Growth in trend TFP using the HP filter is virtually identical, averaging 2.8, 1.6 and 0.1 per cent over the first, second and third break point periods, respectively.

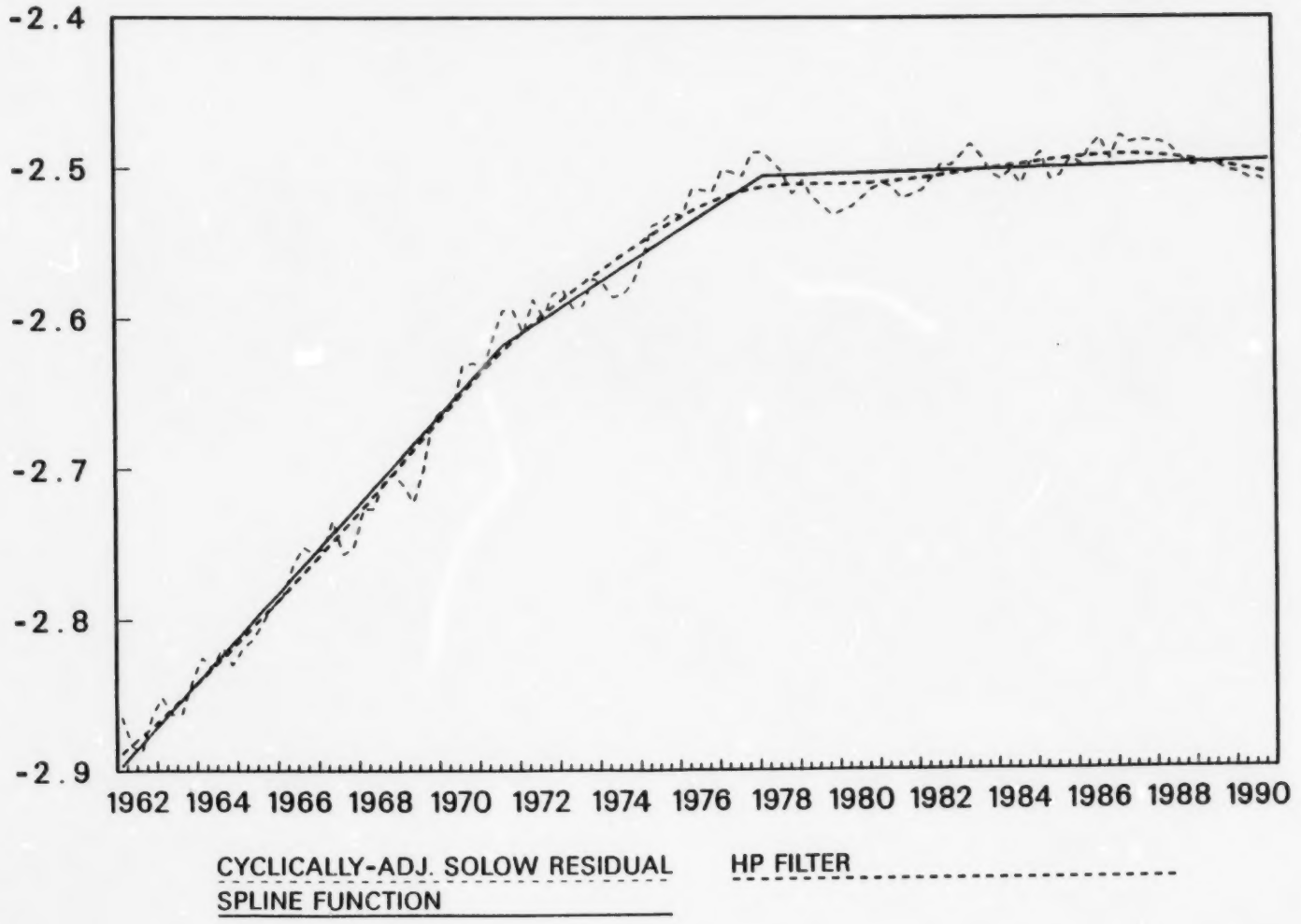
It is evident from these exercises that the estimates of trend TFP growth are robust to the depreciation and break point assumptions. The choice of capital stock data, however, does affect the quantitative (but not qualitative) results. Other researchers employing alternative methodologies obtain different results. In the next section, the methodologies used by other researchers are reviewed in order to pinpoint the source of these differences.

4. Comparison with Other Estimates

This section compares the Finance estimates with those of the OECD, the Economic Council of Canada, and Statistics Canada. Given the wide range of choices required in constructing a measure of total factor productivity, it is not surprising that these estimates differ. Qualitatively, however, all of the studies

16. The HP filter is applied to the cyclically-adjusted Solow residual with the "stiffness" parameter set at the standard value (1600). The similarity with the spline function-based series is not surprising, given that the spline function itself is equivalent to the HP filter under particular parameter choices.

CHART 3.4
SPLINE-BASED TREND TFP
AND HP FILTER-BASED TREND TFP



reveal the same pattern of a significant slowdown in the growth of the Solow residual since the 1960s.

4.1 Comparison of Methodologies

Table 4.1 summarizes the approaches taken by the OECD, the Economic Council, Statistics Canada, and the Department of Finance. All four measure growth in the Solow residual by subtracting the contributions of growth in inputs from growth in output using an assumed production function. The main methodological differences arise from data and parameterization choices. The approaches taken by the Economic Council and Statistics Canada are constrained by their additional need to calculate the Solow residual for specific industry groups.

Statistics Canada's methodology differs more fundamentally from the others in one key respect. Specifically, they use an innovative approach in which TFP (or "multi-factor" productivity) is calculated for each of many input-output industries by subtracting the growth in all intermediate and factor (labour and capital) inputs from growth in industry gross output (the other three estimates are based on a "value-added" or net measure of output). They then obtain an estimate of TFP for the business sector by aggregating the detailed industry residuals using input-output coefficients.[17] This approach permits extensive analysis of the sources of changes in productivity either by specific industry or commodity, thereby identifying the source of the productivity slowdown in a way that is difficult with aggregate approaches. Nonetheless, Statistics Canada still considers their results to be at an experimental stage.

4.2 Comparison of Results

Chart 4.1 and Table 4.2 show the four different estimates. Among the four studies, the Department of Finance is unique in applying a regression approach to the Solow residual in order to identify underlying trends. In order to provide consistent comparisons, the Finance estimates used here are for the growth in the Solow residual only, not the trend. Recall from Table 3.1 that the average growth of the Department of Finance Solow residual varies slightly from trend TFP growth (identified from the spline function) over the three key break periods.

As shown in Chart 4.1, all estimates of the Solow residual follow a broadly similar pattern until the end of the 1970s. The estimates for Statistics Canada, the

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17. More precisely, the business sector TFP can be disaggregated into four components: a weighted sum of industry TFPs and three terms reflecting sectoral reallocations of value added, capital and labour.

Table 4.1
Summary of TFP Methodologies

	OECD	Economic Council of Canada	Statistics Canada	Department of Finance
Output (Source)	Total Economy GDP Excluding Value Added by General Government (OECD Database)	Business Sector GDP (Statistics Canada Input-Output Division)	Business Sector GDP (Aggregate Productivity Measures)	Commercial Sector GDP (Canadian Income and Expenditure Accounts)
Capital Input (Source)	Net Capital Stock Delayed Depreciation? No Capacity Utilization Adjustment (OECD database)	Net Capital Stock Geometric Depreciation No Capacity Utilization Adjustment (Fixed Capital Flows & Stocks)	Net Capital Stock Geometric Depreciation No Capacity Utilization Adjustment (Fixed Capital Flows & Stocks)	Net Capital Stock Delayed Depreciation Capacity Utilization Adjustment (Constructed using a Flow of Investment from the Canadian Income and Expenditure Accounts)
Labour Input (Source)	Employment (OECD Database)	Employment (Labour Force Survey and Survey of Employment, Payroll and Hours)	Employment (Labour Force Survey and Survey of Employment, Payroll and Hours)	Total Hours Worked (Labour Force Survey)
Factor Share	Fixed Weights: Labour Share 0.65 Capital Share 0.35	Time-varying Weights	Törnqvist Weights	Fixed Weights: Labour Share 0.637 Capital Share 0.363
Trend Methodology	Residual	Residual	Residual	Regression of Spline Function Using Observed Turning Points
Level of Aggregation	Aggregate business sector	Aggregate business sector	Aggregation of detailed industry TFPs	Aggregate commercial sector
Publication Source	Englander and Mittelstadt (1988) and unpublished data from OECD Secretariat	Rao & Lemprière (1990) and unpublished data from authors	Aggregate Productivity Measures (15-204)	

CHART 4.1
SOLOW RESIDUAL INDICES

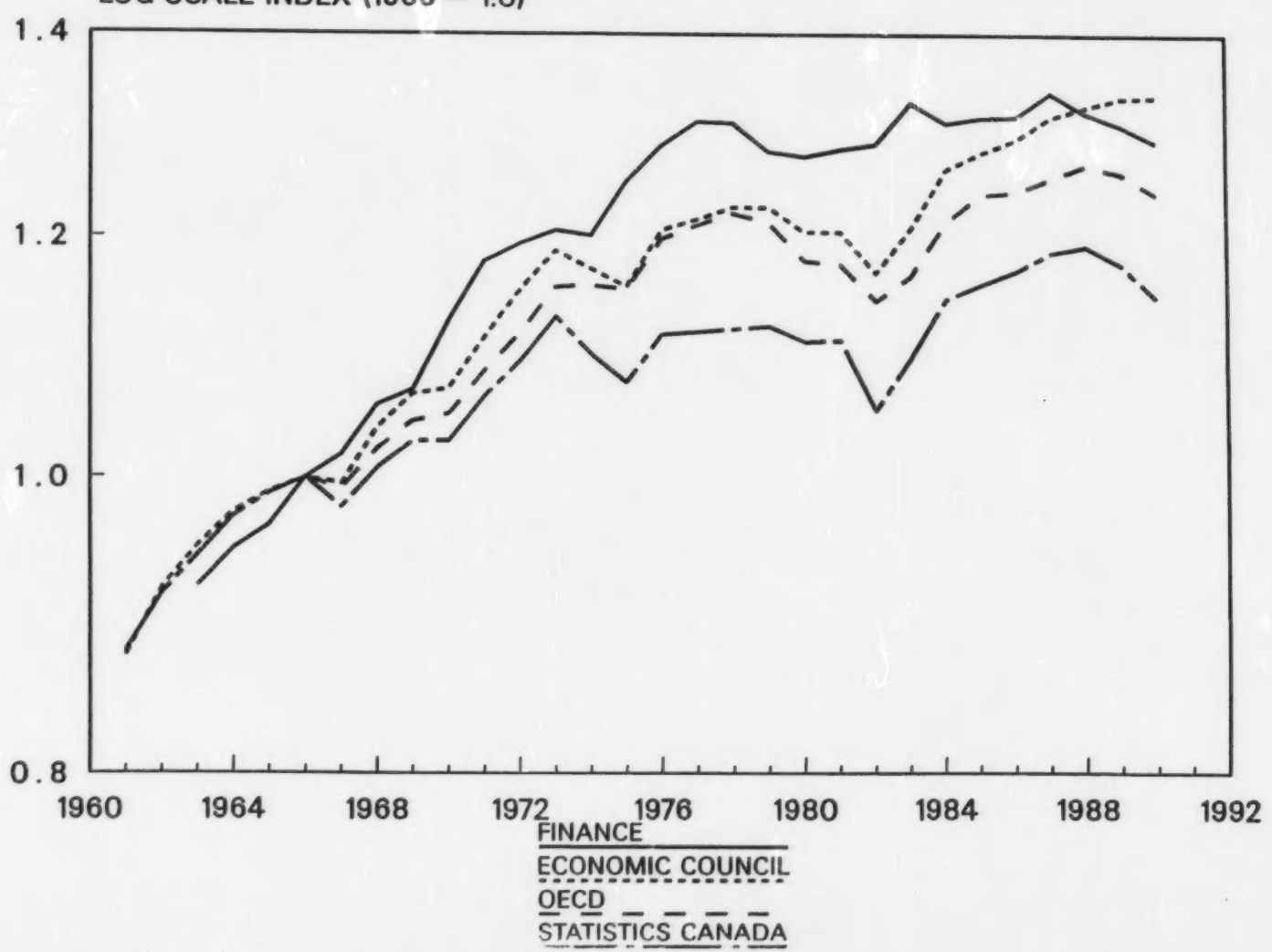

Table 4.2
Average Growth Rates of the Solow Residual, Output and Measured Capital and Labour Inputs

	Major Break Periods[1]			Third Period Subperiods	
	1964-71	1972-77	1978-90	1978-84	1985-90
Solow Residual					
Department of Finance[2]	3.1	1.9	-0.1	0.0	-0.1
OECD[3]	N.A.	1.9	0.2	0.0	0.3
Economic Council	2.0	1.5	0.7	0.6	0.9
Statistics Canada[4]	1.5	0.8	0.2	0.4	0.0
Output					
Department of Finance	5.6	5.4	3.0	2.7	3.4
OECD[3]	N.A.	5.3	3.2	3.0	3.4
Economic Council	5.3	4.9	3.2	2.8	3.7
Capital					
Department of Finance	4.5	6.0	4.7	4.6	4.5
OECD[3]	N.A.	4.8	4.4	5.0	4.4
Economic Council	4.8	3.9	3.5	3.9	3.1
Labour					
Department of Finance	1.3	2.0	2.2	1.5	3.0
OECD[3]	N.A.	2.7	2.1	1.9	2.4
Economic Council[5]	2.4	3.1	1.9	1.3	2.5

(1) The averaging periods are chosen to correspond to the break points used in this study.
(2) Cyclically-adjusted Solow residual.
(3) For the OECD, the data starts in 1967.
(4) Owing to the disaggregated approach adopted by Statistics Canada, only their Solow residual results can be reported.
(5) The Economic Council's labour input for the 1987-90 period was produced from using employment growth rates derived from the labour force survey for the business sector.

part of the difference in the Economic Council and Finance estimates.[19] But the difference between the Economic Council and Finance estimates becomes more

19. As mentioned in section 2.2.c, Finance constructs its capital stock series using investment flows data from the CIEA. The other three studies use a capital stock series constructed from FCFS. Recall that real investment in machinery and equipment is much higher in the FCFS than in the CIEA prior to 1980, although the measures are quite similar thereafter. As a result, the capital stock measure constructed using CIEA investment data grows faster than a stock constructed using FCFS.

significant for the 1985-1990 period. During this period, slower growth in the Economic Council's labour input reinforces the effect of slower growth in the Economic Council's capital input on their Solow residual growth. Much of the difference reflects the rebounds in average weekly hours and capacity utilization which are captured in the Finance measure.

Finance also obtains higher Solow residual growth than Statistics Canada in the first two break periods and less growth in the last break period. While Statistics Canada's methodology differs considerably from that of Finance, differences in measurement of the capital and labour inputs are likely to contribute to the divergence in the estimates. As with the Economic Council, Statistics Canada uses employment rather than hours as the labour input. This would overestimate growth in labour and underestimate growth in the Solow residual relative to Finance in the 1960s and 1970s due to declining average hours. As well, Statistics Canada uses the Fixed Capital Flows and Stocks estimates of the capital stock. Since these grow more slowly than the Finance measure in the third break period, this could account for part of Statistics Canada's higher Solow residual growth. Aggregating Solow residuals by industry to obtain a business sector total may further contribute to the difference.[20]

Statistics Canada uses time-varying, or "Törnqvist", weights for factor shares, while the Economic Council uses annual time-varying weights. These alternative weighting choices do not play a significant role in explaining the estimated Solow residual growth differences.

In the final analysis, it remains difficult to fully account for the differences in the TFP measures. This suggests a need to explore further the sources of the differences in the individual series, particulary the capital stock. Nevertheless, it is possible to draw some basic conclusions. The Finance estimates show the largest dropoff in Solow residual growth over the major subperiods. This is because Finance obtains the highest growth in the first subperiod, since it accounts for the declining in average weekly hours, and the lowest growth in the most recent period, owing to a higher Finance measure of capital stock growth. In the most recent period, 1985 to 1990, the Economic Council and OECD estimates show a rebound in TFP growth. This rebound is not evident in the Finance estimates because it reflects, in part, an increase in average weekly hours and in capacity utilization which is captured in the Finance estimate of factor inputs.

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20. Jorgenson (1988).

5. Comparisons with the United States

International comparisons of TFP growth can be useful in assessing a country's relative performance. These comparisons must be viewed with some caution, however, as differences in data used can significantly bias the results. The following Canada-U.S. comparisons of trend TFP growth are based on the Department of Finance estimates using identical methodologies. Details of estimation of trend TFP growth in the United States are presented in Annex G.

Table 5.1
Comparisons of Trend Total Factor Productivity Growth in Canada and the United States[1]
(average annual per cent change)

	Canada	United States
1963:1-1971:3	2.9	1.1
1971:4-1978:1	1.7	0.2
1978:2-1990:4	0.1	0.5

(1) Averaging periods chosen to correspond with break points for Canada.

Estimates show that trend TFP growth declined during the 1970s in both Canada and the United States. Trend TFP growth is estimated to have declined from 2.9 per cent in Canada over the 1963-1971 period to 1.7 per cent over the 1972-1978 period (Table 5.1). The U.S. also experienced a decline in trend TFP growth over this period, from 1.1 per cent to 0.2 per cent. Although researchers have suggested various factors that have contributed to these losses -- including energy price shocks, declining investment in research and development, measurement problems and growth of the service sector -- the exact contribution of each is not clear.[21] An exhaustive analysis of this issue is beyond the scope of this paper.

In sharp contrast to the Canadian experience, trend TFP growth appears to have improved in the United States in the 1980s. Over the late 1970s and 1980s, trend TFP growth in the United States is estimated to have risen to an average of

21. See for, example, Baily and Gordon (1989), Denison (1989), Jorgenson (1988) and Griliches (1988).

0.5 per cent.[22] This stands in sharp contrast to the Canadian experience where trend TFP growth is estimated to have declined to 0.1 per cent per year over the same period. A factor that may have contributed to the U.S. recovery is the strong productivity performance of the computer industry (Denison (1989)). This performance reflects, in part, the inclusion of improvements in computing power in the measure of output. Since this industry has a much smaller share in production in Canada, it cannot make the same contribution to aggregate productivity growth.[23] Differences in industrial structure may therefore explain part of the divergence as may more fundamental differences in behaviour.

6. Summary and Conclusions

TFP growth is a key determinant of the rate at which an economy can provide its citizens with improved living standards. Using a standard methodology, the Department of Finance estimates that trend TFP growth has declined from 2.9 per cent over the 1963 to 1971 period, to 1.7 per cent over the 1972 to 1978 period, and to 0.1 per cent over the 1979 to 1990 period. By comparison, the other studies examined generally estimate lower TFP growth in the 1960s and 1970s, mainly because of higher growth in the labour input. They estimate higher TFP growth in the 1980s, because of slower growth in both inputs. The fact that these studies do not adjust the capital stock for capacity utilization may also explain at least part of their estimate of an increase in TFP growth in the latter part of the 1980s. Nevertheless, these studies support the key qualitative conclusion of a significant slowdown in Canadian trend TFP growth in the 1970s and the 1980s.

The estimates discussed here reflect a series of judgements regarding the best choice of output and factor input measures (with respect to both concept and choice of data source) and the aggregate technology relationship. Measuring the labour input as total hours worked, rather than employment, has the advantage of capturing the declining trend in average hours worked per week through the 1960s and 1970s, which would otherwise be attributed to a lower estimated Solow residual growth rate during this period. It may also capture some cyclical fluctuations in work intensity. Since TFP is typically interpreted as a measure of the general efficiency of resource allocation and of production technology, it should not be directly affected by developments such as a changing work week.

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22. This pattern was found by other authors. See, for example, Baumol et al. (1990) and Adams and Coe (1989).

23. Denison (1989) and Baily and Gordon (1988), however, suggest that technical price indices used to calculate real output measures may have overstated output growth in the computer industry.

The capacity utilization adjustment made to capital allows underlying trends in the Solow residual to be more easily separated from cyclical fluctuations. The spline function approach isolates periods of constant growth in trend TFP and has the advantage of considering all of the data throughout the sample period.

Finance estimates of trend TFP growth are robust to the choice of break points, the capital depreciation assumption and the capacity utilization adjustment to capital. Although the qualitative results are unchanged, using FCFS capital stock data generates somewhat higher trend TFP growth estimates in the 1970s and 1980s. This is due to higher CIEA estimates of growth in machinery and equipment investment prior to 1980.

While other defensible choices can be made, the choice and refinement of data underlying the Finance estimates are appropriate for estimating trend TFP growth for the commercial sector of the economy. In general, however, these choices do not seem to change the key qualitative conclusion of a significant slowdown in Canadian trend TFP growth in both the 1970s and again in the 1980s.

There are a number of issues which merit further examination and research. First, although this study shows that growth in the cyclically-adjusted Solow residual appears to be uncorrelated with growth in a number of exogenous demand impulse variables, the assumptions underlying the invariance properties of the residual should be tested directly. Specifically, the validity of the constant returns to scale and perfect competition assumptions and the impact of altering these assumptions on the results should be determined. Second, the potential advantages of adopting a disaggregated approach would be worth exploring. In that respect, the Statistics Canada disaggregated approach could yield some important insights.

A final and critical point is that measurement is not explanation. The fundamental sources of the productivity slowdown are not fully understood. An econometric study of the causes of the estimated slowdown in Canadian TFP growth would be particularly useful in determining the appropriate policy response. The sources of the rebound in trend TFP growth in the United States in the 1980s would also be useful in this regard.[24] In such a study, particular attention would be paid to the role of research and development expenditures, new patents, the growth of the service sector, and energy price shocks.[25]

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24. Some studies indicate that this may be due in part to the phenomenal growth in the productivity of the U.S. computer industry, an industry too small in Canada to have made any contribution to Canadian aggregate TFP growth.

25. The Economic Council of Canada has already done some of this econometric analysis and has had some success at trying to explain the sources of the Canadian productivity slowdown. See Rao and Lemprière (1991).

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Annex A
Income Share of Capital Types

The capital stock, K, is measured as a bundle of machinery and equipment, K_{me}, and non-residential structures, K_{nr}:

$$K = K_{me}^{\beta} K_{nr}^{(1-\beta)} \qquad (A.1)$$

The parameter β is estimated by calculating the implicit machinery and equipment income share in total capital income:

$$\beta = \text{Sample Average } \{U_{me}K_{me}/(U_{me}K_{me} + U_{nr}K_{nr})\} \qquad (A.2)$$

where U_{me} and U_{nr} are the user costs for machinery and equipment and non-residential structures, respectively. These are calculated using the standard formulae derived from firm value maximization conditions:

$$U_i = (1 - \theta_i)(P_i/P_Y)(\gamma + \pi_Y^e - \pi_i^e + \delta_i)$$

$$[1 - \tau_c\delta_i/(\gamma + \delta_i + \pi_i^e)]/(1 - \tau_c) \quad , i = me, nr \qquad (A.3)$$

where:

θ_i is the rate of investment tax credit for capital type i
P_i is the investment price deflator for capital type i
P_Y is the output deflator, γ is the real cost of funds[26]
π_Y^e is the expected rate of inflation[27]
π_i^e is the expected rate of price inflation for capital type i
δ_i is the depreciation rate for capital type i, and
δ_i is the capital consumption allowance rate for capital type i.
τ_c is the corporate tax rate
γ is the real cost of funds
The estimate for β over the sample period is 0.4.

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26. The real cost of funds is measured by weighting equity and debt financing costs by the equity and debt financing shares. For a detailed discussion of the methodology see James (1991).

27. Expected inflation follows an adaptive expectations formulation.

Annex B
Investment Flows: Canadian Income and Expenditure Accounts and Fixed Capital Flows and Stocks

Investment in fixed capital in both the Canadian Income and Expenditure Accounts (CIEA) and the Fixed Capital Flows and Stocks (FCFS) is based on Statistics Canada's capital expenditure surveys on Private and Public Investment Intentions (PPI). As mentioned in section 2.2.c, divergences occur between these two sources in both the nominal and real data.

The differences in the nominal investment data are small, and result primarily from CIEA definitional adjustments to the PPI survey data (eg. real estate commissions are included in CIEA estimates) that are not adopted by the FCFS (Charts B.1 and B.2)

The differences in the real investment data are much more pronounced, especially for machinery and equipment (Charts B.3 and B.4). They result primarily from different types of aggregation used in the deflation process. Specifically, CIEA investment is deflated by commodity, while FCFS investment is deflated by industry. This results in a different aggregate real investment series.[28] The divergence is compounded by the 1986 rebasing exercise since the CIEA used adjusting entries to preserve growth rates of the data by commodity, while the FCFS used adjusting entries to preserve the growth rates of the data by industry. This results in divergences in the two series which are greater the further the data are from the base year.

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28. In order to be more consistent with the CIEA, the FCFS changed to a Paashe-type price index using industry and commodity weights from Statistics Canada's input-output model. The FCFS machinery and equipment data were revised back to 1961 using this new index. The FCFS non-residential construction data were revised back to 1981. Prior to this base-year weights are used. For further details see Statistics Canada (1990).

CHART B.1
NOMINAL INVESTMENT IN MACHINERY AND EQUIPMENT
(NATURAL LOG)

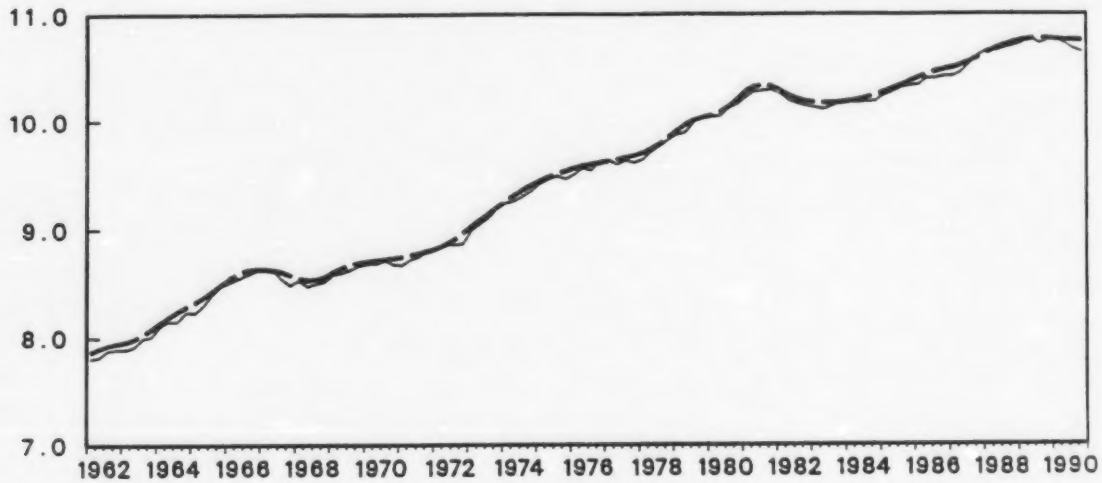


CHART B.2
NOMINAL INVESTMENT IN NON-RESIDENTIAL CONSTRUCTION
(NATURAL LOG)

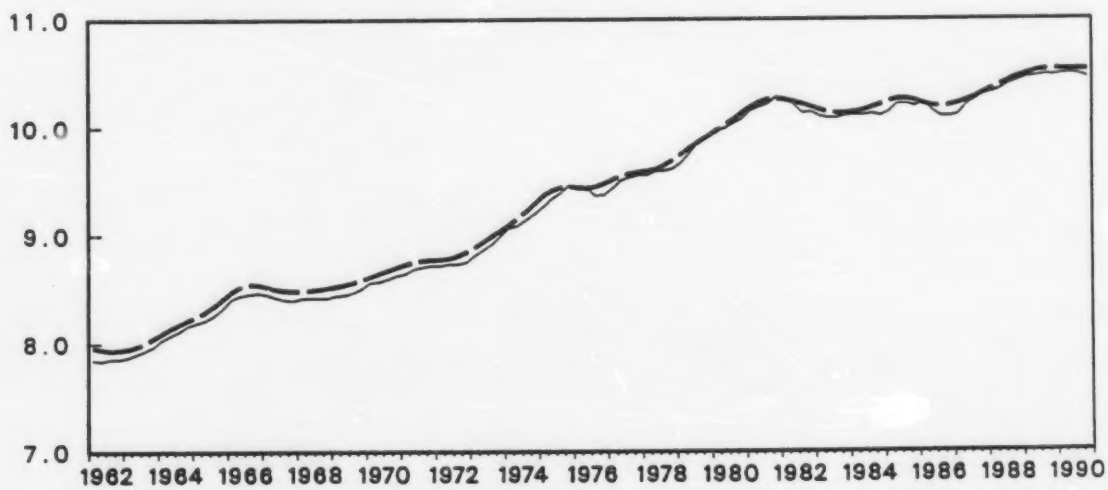


CHART B.3
REAL INVESTMENT IN MACHINERY AND EQUIPMENT
(NATURAL LOG)

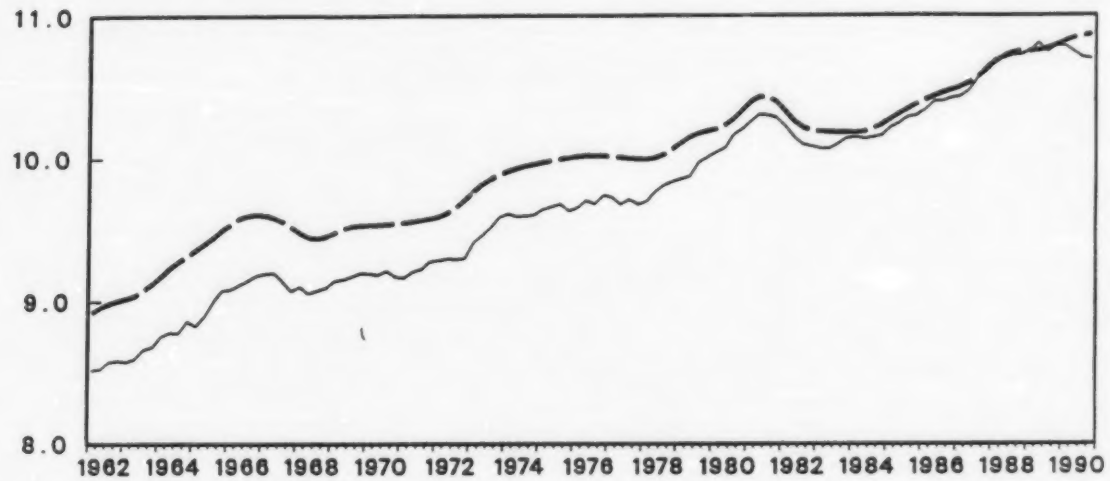
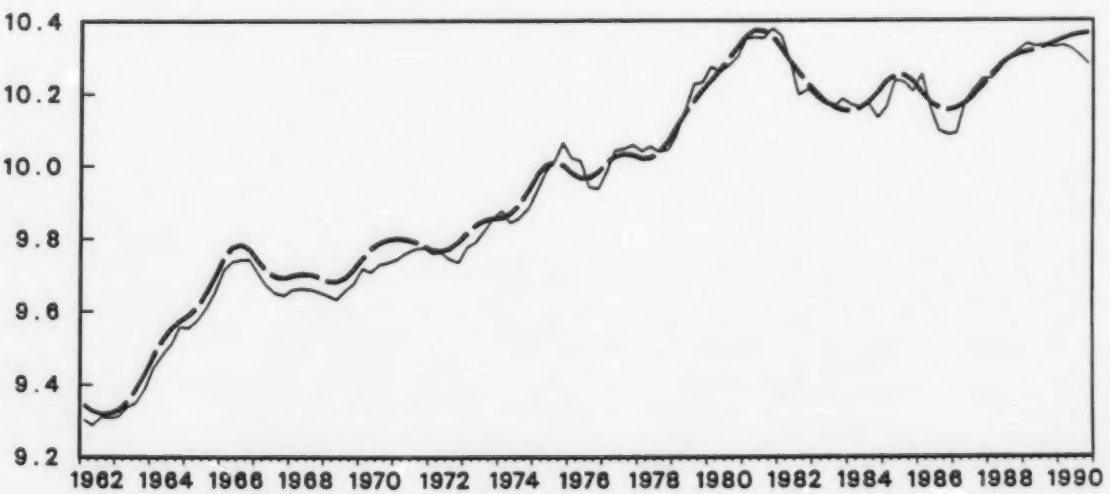


CHART B.4
REAL INVESTMENT IN NON-RESIDENTIAL CONSTRUCTION
(NATURAL LOG)



Annex C
The Capacity Utilization Index

The capacity utilization index uses the Bank of Canada's index for all goods-producing industries as a base and adjusts it to account for the lower variability of energy and service industries over the business cycle. Recall from Annex A that the capital input is measured as the weighted sum of capital for machinery and equipment, K^{me}, and non-residential construction, K^{nr}, for both goods and services industries:

$$K = K_{me}^{\beta} K_{nr}^{(1-\beta)} \qquad (C.1)$$

where

$$K_i = K_i^g + K_i^s \qquad , i = me, nr \qquad (C.2)$$

and g is capital in goods-producing industries, and s is capital in service and energy-producing industries.

The capacity utilization index is equal to:

$$\lambda = \theta\lambda^g + (1-\theta)\lambda^s \qquad (C.3)$$

where θ is the share of total capital in use in the goods producing industries, λ^g is equal to the Bank of Canada's capacity utilization index for all goods-producing industries excluding energy (normalized on its 1965:1 to 1969:4 average) and λ^s is derived as:

$$\lambda^s = \delta/\hat{\delta} \qquad (C.4)$$

where

$$\delta = [\varphi\lambda^{g'} + (1-\varphi)\hat{\lambda}^{g'}] \qquad (C.5)$$

$\lambda^{g'}$ is the unadjusted Bank of Canada's series, $\hat{\lambda}^{g'}$ is the mean of $\lambda^{g'}$, $\hat{\delta}$ is the mean of δ, and φ is ratio of the variances of K^s and K^g relative to output in each respective sector. This method simply reduces the amplitude of the capacity utilization index without changing its average.

ORDINARY LEAST SQUARES

MODEL NAME: TFPMOD

2 : TFPR = B0+B1*TIME*DUMMY1+((B1-B2)*39+B2*TIME)*DUMMY2+((B1-B2)*39+(B2-B3)*65+B3*TIME)*DUMMY3

NOB = 112		NOVAR = 4		NCOEF = 4		RANGE: 1962 1	TO 1989 4
RSQ =	0.989404	CRSQ =	0.98911	F(3/108) =	3361.59	PROB>F =	0.
SER =	0.013275	SSR =	0.019033	DW(0) =	0.656839	COND =	9.98381
MAX:MAT =	0.0892	RSTUDENT =	-3.06266	DFFITS =	0.840827	LHS MEAN =	-2.60264
SUMR =	-9.865442E-13	RSQ* =	0.999975				

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
B0	-2.90437	0.004102	-708.006	0.
B1	0.007384	0.000157	47.0361	0.
B2	0.004236	0.000186	22.759	0.
B3	0.000307	0.000118	2.6037	0.010519

COEF	MEAN	STDEV	PARTIAL	BETA
B0	1.	0.	0.	0.
B1	32.3839	11.2772	0.976451	0.654628
B2	14.0446	11.9612	0.909653	0.398342
B3	10.0714	14.814	0.243029	0.035789

Annex E
Solow Residual Invariance Tests

As mentioned in Section 2, the Cobb-Douglas production function used imposes constant returns to scale in factor inputs. It is also assumed that all input prices are determined competitively and that all firms choose their inputs so as to maximize profit.

Hall (1990) points out that these assumptions imply that the Solow residual should be uncorrelated with any variable that would be uncorrelated with "true" TFP if the latter could be directly observed. He tests for this desired invariance property by regressing growth in his estimated Solow residual upon growth in a number of exogenous demand impulse variables which are correlated with output. The instruments chosen are real military expenditures, the political party in control of the presidency and oil price shocks. From his regression estimates he concludes that in general the hypothesis of invariance is rejected.

Since Canada is a small open economy, the U.S. and export share-weighted G-7 output growth rates are a natural choice of exogenous instruments for conducting invariance tests upon the estimated Canadian Solow residual. Canadian output growth should be positively correlated with growth in these instruments, which should be essentially unaffected by developments within Canada.

Table E.1 reports the results of tests for correlation between growth in real Canadian commercial sector output and growth in the instruments, while Tables E.2 and E.3 report the results of the invariance tests themselves. To confirm the robustness of the results, current and lagged values of the quarterly growth rates of both instruments are used and the tests are conducted over three periods; 1967 to 1990, 1967 to 1977 and 1978 to 1990.

Table E.1 reveals that, as expected, growth in real Canadian commercial sector output is positively correlated with growth in the instruments. With the exception of the choice of U.S. output as an instrument over the period 1967 to 1977, the hypothesis of a positive correlation cannot be rejected at the 95 per cent confidence level. The variables chosen are thus appropriate for use as exogenous instruments in the invariance tests.

Tables E.2 and E.3 show that in all cases the hypothesis of invariance between growth in the cyclically-adjusted Solow residual and growth in the

instruments cannot be rejected at the 95 per cent confidence level. This is in significant contrast to Hall's results for the United States. Except for the period 1967 to 1977, however, the invariance hypothesis can be rejected for the raw (without capacity adjustment) Solow residual. Hall found that a capacity adjustment to capital did not alter his rejection of the invariance hypothesis; however, such an adjustment does appear to be key to the acceptance of invariance of the Canadian Solow residual.

These results add support to the method used in this study to calculate the cyclically-adjusted Solow residual. In order to fully test the methodology, however, direct tests of the assumptions (perfect competition and constant returns to scale) are required.

Table E.1
Regression Tests of Correlation Between Innovations in Canadian Commercial Sector Output and the Exogenous Instruments

Regression:	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
Regressor	ΔY	ΔY	ΔY	ΔY	ΔY	ΔY	ΔY	ΔY
Bounds	67-90	67-77	78-90	70-90	67-90	67-77	78-90	70-90
Constant	0.006	0.010	0.0024	0.0047	0.007	0.009	0.004	0.0049
se	0.0015	0.0026	0.0016	0.0018	0.002	0.003	0.002	0.0018
t	4.29	4.25	1.60	2.63	4.29	3.78	0.44	2.27
ΔY_{US}	0.46	0.23	0.69					
se	0.13	0.22	0.13					
t	3.65	1.07	5.15					
ΔY_{G7}				0.685				
se				0.171				
t				4.00				
ΔY_{US-1}					0.459	0.45	0.46	
se					0.129	0.21	0.15	
t					3.58	2.15	3.07	
ΔY_{G7-1}								0.66
se								0.17
t								3.79
R^2	0.13	0.026	0.352	0.168	0.121	0.099	0.161	0.154
DW	2.12	2.42	1.99	2.17	2.15	2.52	2.40	1.97

Data definitions:

ΔY = quarterly growth rate of real Canadian commercial sector output
ΔY_{US}, ΔY_{US-1} = quarterly current and lagged growth rates of real U.S. GDP
ΔY_{G7}, ΔY_{G7-1} = quarterly current and lagged growth rates of real Canadian export share-weighted G7 output

Table E.2
Regression Tests of Solow Residual Invariance Properties

Regression:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Regressor	$\Delta\hat{Q}$	$\Delta\hat{Q}$	$\Delta\hat{Q}$	$\Delta\hat{Q}$	ΔQ	ΔQ	ΔQ	ΔQ
Bounds	67-90	67-77	78-90	70-90	67-90	67-77	78-90	70-90
Constant	0.0006	0.005	-0.003	-0.0008	0.004	0.007	0.0003	0.003
se	0.0015	0.0025	0.0015	0.0017	0.0013	0.0023	0.0013	0.0014
t	0.43	1.99	-2.22	-0.505	2.68	3.16	0.233	2.00
ΔY_{US}	0.267	0.133	0.391		-0.142	-0.188	-0.190	
se	0.125	0.212	0.125		0.112	0.193	0.110	
t	*2.14*	**0.627**	*3.12*		**-1.27**	**-0.978**	**-0.909**	
ΔY_{G7}				0.397				-0.145
se				0.161				0.137
t				*2.46*				**-1.05**
R^2	0.047	0.009	0.166	0.071	0.017	0.022	0.017	0.014
DW	2.10	2.31	2.16	2.11	2.12	2.33	2.29	2.09

A t-statistic in bold indicates that the hypothesis of invariance cannot be rejected at the 95 per cent confidence level. A t-statistic in italics indicates that the hypothesis of invariance is rejected at the 95 per cent confidence level.

Data definitions:

$\Delta\hat{Q}$ = quarterly growth rate of the raw Solow residual
ΔQ = quarterly growth rate of the cyclically-adjusted Solow residual
ΔY_{US} = quarterly growth rate of real U.S. GDP
ΔY_{G7} = quarterly growth rate of real Canadian export share-weighted G7 output

Table E.3
Regression Tests of Solow Residual Invariance Properties

Regression:	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Regressor	$\Delta\hat{Q}$	$\Delta\hat{Q}$	$\Delta\hat{Q}$	$\Delta\hat{Q}$	ΔQ	ΔQ	ΔQ	ΔQ
Bounds	67-90	67-77	78-90	70-90	67-90	67-77	78-90	70-90
Constant	0.010	0.004	-0.002	-0.0002	0.003	0.006	0.0005	0.0027
se	0.0015	0.0025	0.002	0.0017	0.001	0.002	0.001	0.0014
t	0.69	1.63	-1.03	-0.096	2.46	2.76	0.44	1.90
ΔY_{US-1}	0.21	0.27	0.144		-0.101	-0.067	-0.14	
se	0.13	0.21	0.14		0.11	0.196	0.11	
t	**1.64**	**1.27**	**1.06**		**-0.90**	**-0.34**	**-1.29**	
ΔY_{G7-1}				0.297				-0.124
se				0.164				0.138
t				**1.80**				**-0.90**
R^2	0.028	0.037	0.166	0.040	0.009	0.013	0.033	0.010
DW	2.03	2.33	2.16	1.95	2.15	2.30	2.40	2.15

A t-statistic in bold indicates that the hypothesis of invariance cannot be rejected at the 95 per cent confidence level. A t-statistic in italics indicates that the hypothesis of invariance is rejected at the 95 per cent confidence level.

Data definitions:

$\Delta\hat{Q}$ = quarterly growth rate of the raw Solow residual
ΔQ = quarterly growth rate of the cyclically-adjusted Solow residual
ΔY_{US-1} = lagged quarterly growth rate of real U.S. GDP
ΔY_{G7-1} = lagged quarterly growth rate of real Canadian export share-weighted G7 output

ORDINARY LEAST SQUARES

MODEL NAME: TFPMOD

4 : TFPR = B0+B1*TIME+DUMMY1+((B1-B2)*39+B2*TIME)*ALT22+((B1-B2)*39+(B2-B3)*61+B3*TIME)*ALT23

NOB = 112	NOVAR = 4	NCOEF = 4		RANGE: 1962 1	TO 1989 4
RSQ = 0.990579	CRSQ = 0.990317	F(3/108) = 3785.19		PROB>F = 0.	
SER = 0.012518	SSR = 0.016923	DW(0) = 0.740481		COND = 10.2222	
MAX:HAT = 0.090082	RSTUDENT = -3.07493	DFFITS = 0.855584		LHS MEAN = -2.60264	
SUMR = -1.463274E-12	RSQ* = 0.999978				

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
B0	-2.90269	0.003889	-746.393	0.
B1	0.007258	0.000151	48.0459	0.
B2	0.004858	0.000209	23.2404	0.
B3	0.000452	0.000101	4.45685	2.040749E-05

COEF	MEAN	STDEV	PARTIAL	BETA
B0	1.	0.	0.	0.
B1	32.3839	11.2772	0.977397	0.643466
B2	12.2768	10.222	0.912887	0.390381
B3	11.8393	16.3925	0.394144	0.058239

ORDINARY LEAST SQUARES

MODEL NAME: TFPMOD

3 : TFPR = B0+B1*TIME+DUMMY1+((B1-B2)*39+B2*TIME)*ALT12+((B1-B2)*39+(B2-B3)*69+B3*TIME)*ALT13

NOB = 112		NOVAR = 4		NCOEF = 4		RANGE: 1962 1	TO 1989 4
RSQ =	0.986625	CRSQ =	0.986253	F(3/108) =	2655.5	PROB>F =	0.
SER =	0.014915	SSR =	0.024026	DW(0) =	0.527925	COND =	9.76656
MAX:MAT =	0.088445	RSTUDENT =	-2.89721	DFFITS =	0.785473	LHS MEAN =	-2.60264
SUMR =	-8.320011E-13	RSQ* =	0.999968				

COEF	ESTIMATE	STER	TSTAT	PROB>|T|
B0	-2.90647	0.004588	633.548	0.
B1	0.007542	0.000173	43.5375	0.
B2	0.003688	0.00018	20.4819	0.
B3	0.000214	0.000147	1.45675	0.148085

COEF	MEAN	STDEV	PARTIAL	BETA
B0	1.	0.	0.	0.
B1	32.3839	11.2772	0.972674	0.668598
B2	15.6696	13.6411	0.891776	0.395439
B3	8.44643	13.2351	0.130819	0.022272

Annex G
U.S. Trend TFP Growth

Trend TFP growth for the United States is estimated using both the spline function and Hodrick-Prescott filter. The level of aggregation is commercial output, the labour input is measured in hours worked and the capital input is adjusted for capital utilization[29]. All series are consistent with the revised 1987-based US data.

Examining the Solow residual indicates four break points: the first quarter of 1969; the second quarter of 1973; the first quarter of 1982; and the first quarter of 1986. Trend TFP is calculated by fitting a spline function to the Solow residual using these break points (Chart G.1). As an alternative, the HP filter is also fitted to the Solow residual. Trend TFP growth over these periods is shown in Table G.1. Trend TFP growth estimates using the two smoothing methods are broadly similar.

Table G.1
Trend TFP Growth in the United States
(average annual per cent change)

	Spline Function	HP Filter
1969:1-1973:1	0.9	0.8
1973:2-1981:4	0.0	0.1
1982:4-1985:4	1.2	0.9
1986:1-1991:4	0.5	0.6

The appropriate measurement of the flow of productive services provided by the capital stock involves issues such as identifying quality improvements in computers and application of the proper depreciation method. Oliner (1989) advocates the use of the Bureau of Labor Statistics (BLS) capital input series for studying productivity. Because they are maintained and updated regularly by DRI, this study uses Bureau of Economic Analysis (BEA) capital stock data, which Oliner considers to be less appropriate[30]. If the mismeasurement of either capital or labour is constant, however, the estimate of trend TFP growth should not be biased.

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29. A capacity utilization rate series for manufacturing industries from Data Resources Incorporated (DRI) is scaled to average one over the sample period.

30. The BEA capital stock data represent the cost of purchasing tangible capital and are intended as measures of capital *wealth*. The BLS data series measure the service flow of capital and are intended to be used in analyses of output and productivity.

CHART G.1

SOLOW RESIDUAL AND TREND TFP

NATURAL LOG